

Proceedings  
 Vol. , No. () 1–3  
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## ON THE POSSIBILITY OF MEASURING THE LENSE–THIRING EFFECT WITH A LAGEOS-LAGEOS II-OPTIS MISSION

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The possibility of performing post-Newtonian gravitoelectromagnetic measurements with a joint LAGEOS-LAGEOS II-OPTIS space-based mission is investigated

OPTIS<sup>1</sup> is a recently proposed satellite-based mission which would allow for precise tests of basic principles underlying Special Relativity and post-Newtonian gravity. This mission is based on the use of a spinning drag-free satellite in an eccentric, high-altitude orbit which should allow to perform a three orders of magnitude improved Michelson–Morley test and a two orders of magnitude improved Kennedy–Thorndike test. Moreover, it should also be possible to improve by two orders of magnitude the tests of the universality of the gravitational redshift by comparison of an atomic clock with an optical clock. Since it is not particularly important for the present version of the mission, the final orbital configuration of OPTIS has not yet been fixed; in ref<sup>1</sup> a perigee height of 10000 km and apogee height of 36000 km, with respect to Earth's surface, are provisionally proposed assuming a launch with Ariane 5.

The requirements posed by the drag-free technology to be used, based on the field emission electrical propulsion (FEEP) concept, yield orbital altitudes not less than 1000 km. On the other hand, the eccentricity  $e$  should not be too high in order to prevent passage in the Van Allen belts which could affect the on-board capacitive reference sensor. Moreover, the orbital period  $P_{\text{OPT}}$  should be shorter than the Earth's daily rotation of 24 hours. The orbital configuration proposed in ref<sup>1</sup> would imply a semimajor axis  $a_{\text{OPT}} = 29300$  km and an eccentricity  $e_{\text{OPT}} = 0.478$ . With such values the difference of the gravitational potential  $U$ , which is relevant for the gravitational redshift test, would amount to  $\Delta U/c^2 \sim 1.8 \times 10^{-10}$ . Such result is about three orders of magnitude better than that obtainable in an Earth-based experiment.

An essential feature of OPTIS is the drag-free control of the orbit. For a drag-free motion of the satellite a sensor measuring the actual acceleration and thrusters counteracting any acceleration to the required precision are needed. The sensor,

which is based on a capacitive determination of the position of a test mass, has a sensitivity of up to  $10^{-12} \text{cm s}^{-2} \text{Hz}^{-\frac{1}{2}}$ . Similar drag-free systems of similar accuracy and with mission adapted modifications will be used in MICROSCOPE, STEP and LISA. These systems have a lifetime of many years.

In this paper we wish to investigate the possibility to use the orbital data of OPTIS for performing precise tests of post-Newtonian gravitoelectromagnetism as well<sup>3</sup>. The gravitomagnetic Lense-Thirring effect on the orbit of a test body<sup>4</sup> is given by secular precessions of the longitude of the ascending node  $\Omega$  and the argument of pericentre  $\omega$

$$\dot{\Omega}_{\text{LT}} = \frac{2GJ}{c^2 a^3 (1-e^2)^{3/2}}, \quad \dot{\omega}_{\text{LT}} = -\frac{6GJ \cos i}{c^2 a^3 (1-e^2)^{3/2}}, \quad (1)$$

where  $G$  is the Newtonian gravitational constant,  $J$  is the proper angular momentum of the central mass,  $c$  is the speed of light and  $i$  is the inclination of the orbital plane to the central mass's equator. The gravitoelectric pericentre advance is<sup>5</sup>

$$\dot{\omega}_{\text{GE}} = \frac{3nGM}{c^2 a (1-e^2)}, \quad (2)$$

where  $n = (GM/a^3)^{1/2}$  is the Keplerian mean motion.

The rather free choice of the orbital parameters of OPTIS and the use of a new drag-free technology open up the possibility to extend its scientific significance with new important post-Newtonian tests. Indeed, it would be of great impact and scientific significance to concentrate as many relativistic tests as possible in a single mission, including also measurements in geodesy, geodynamics. Another important point is that OPTIS is currently under serious examination by a national space agency-the German DLR. Then, even if it turns out that OPTIS would yield little or no advantages for the measurement of the Lense-Thirring effect with respect, e.g., to the originally proposed LARES<sup>6,7</sup>, if it will be finally approved and launched it will nevertheless be a great chance for detecting, among other things, the Lense-Thirring effect.

In Table 1 we report the orbital parameters of the existing or proposed LAGEOS-type satellites and of the originally proposed OPTIS configuration.

Table 1. Orbital parameters of LAGEOS, LAGEOS II, LARES and OPTIS

Orbital parameter	LAGEOS	LAGEOS II	LARES	OPTIS
$a$ (km)	12270	12163	12270	29300
$e$	0.0045	0.014	0.04	0.478
$i$ (deg)	110	52.65	70	63.4
$n$ ( $\text{s}^{-1}$ )	$4.643 \times 10^{-4}$	$4.710 \times 10^{-4}$	$4.643 \times 10^{-4}$	$1.258 \times 10^{-4}$

The main characteristics of such a mission are the already mentioned drag-free technique for OPTIS and the Satellite Laser Ranging (SLR) technique for tracking. Today it is possible to track satellites to an accuracy as low as a few mm. This may

be further improved in the next years. With the level of accuracy reached with the most recent, although preliminary, Earth gravity model solutions like GGM01C<sup>a</sup>, a three-nodes combination could be considered. Indeed, by using the nodes of LAGEOS, LAGEOS II (with a coefficient of  $3 \times 10^{-3}$ ) and OPTIS in the LARFES orbital configuration (with a coefficient of  $9.9 \times 10^{-1}$ ) the relative error due to the static part of geopotential, according to the variance matrix of GGM01C (RSS calculation) would be  $3 \times 10^{-5}$ , with a pessimistic upper bound of  $6 \times 10^{-5}$ . The slope of the gravitomagnetic signal would be  $61.4 \text{ mas yr}^{-1}$ . In this case, since the nodes are insensitive to the post-Newtonian gravitoelectric shift which, instead, affects the perigee, the result of such test would be independent of the inclusion of it into the force models. With the three-nodes combination it should not be too optimistic to predict a total error less than 1% over a time span of a few years.

The analysis of the perigee only of OPTIS-in the LARES orbital configuration-could allow to measure the post-Newtonian gravitoelectric shift. Indeed, for the LARES orbital configuration eq.(2) yields a secular advance of  $3280.1 \text{ mas yr}^{-1}$ . This implies that, for  $\delta r^{\text{exp}} \sim 1 \text{ cm}$  over 1 year, the experimental error would be  $\sim 1.3 \times 10^{-3}$ . The impact of the mismodelling in the geopotential, according to the present-day variance matrix of GGM01C, is of the order of  $2 \times 10^{-3}$ , with a pessimistic upper bound of  $4 \times 10^{-3}$  due to the sum of the absolute values of the individual errors. The impact of  $\delta(J_2^{\text{eff}})$  would yield a relative error of  $4 \times 10^{-4}$  over one year. For the non-gravitational perturbations, we could assume<sup>b</sup> an error of  $\sim 6 \times 10^{-4} - 1 \times 10^{-3}$ .

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<sup>a</sup>It can be retrieved on the WEB at <http://www.csr.utexas.edu/grace/gravity/>. The GGM01C model combines the Center for Space Research (CSR) TEG-4 model with data from GRACE only. It seems to be very promising for our purposes. Indeed, the released sigmas are not the mere formal errors but are approximately calibrated.

<sup>b</sup>E.g. for 10-30 perigee perturbations with amplitudes of  $0.2 \text{ mas yr}^{-1}$ . It can be shown using Table 4 of ref<sup>7</sup> that by using only the perigee of LARES the non-gravitational perturbations would have an impact of almost  $4 \times 10^{-3}$  over 7 years.